Second-Order Entanglement Disequilibrium as a Dark-Sector Effective Theory

A relative-entropy response route from Einstein gravity to coupled quintessence and ultralight phase dark matter

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Abstract

We propose a conditional effective-field-theory framework in which the dark sector is the leading long-wavelength response to local entanglement disequilibrium. The construction begins from the established entanglement-equilibrium programme: for small causal diamonds, stationarity of generalized entropy at fixed volume reproduces the semiclassical Einstein equation at first order. We then ask what remains at second order. Positivity of quantum relative entropy implies that the first non-vanishing departure from local entanglement equilibrium defines a positive information metric, equivalent in holographic settings to canonical energy. If the lightest coarse-grained response sector is two-dimensional and carries non-zero modular Berry curvature, the associated quantum geometric tensor defines a natural complex structure. The corresponding low-energy coordinate is therefore a polar response field

$$\Phi_E(x) = \varrho(x)e^{i\theta(x)}.$$

In this minimal sector, the radial mode behaves as dynamical dark energy under slow roll, while the compact angular mode behaves as ultralight, coherently oscillating dark matter after $m_\theta \gtrsim H$. The polar information metric fixes a dark-sector interaction,

$$\dot{\rho}_\theta + 3H\rho_\theta = -\frac{\dot{\varrho}}{\varrho}\rho_\theta,$$

rather than adding a phenomenological coupling by hand. This yields a late-time consistency relation between dark-energy rolling, dark-matter dilution, and phase-mass drift, as well as a pre-inflationary isocurvature bound that drives the full-phase-dark-matter branch to very small primordial tensors, typically $r \lesssim 10^{-5}$ for $m_\theta \sim 10^{-20}\,\text{eV}$. We do not claim a completed microscopic theory of quantum gravity. The core claim is narrower: if Einstein gravity is first-order entanglement equilibrium, then a two-dimensional second-order modular response sector naturally produces a coupled radial/phase dark-sector EFT. The framework is falsifiable through the tensor-to-scalar ratio, isocurvature, the time dependence of dark energy, ultralight-dark-matter structure suppression, and any confirmed non-gravitational phase portal such as cosmic birefringence.

Contents

1 Status and central claim

This manuscript is a sharpened replacement for an earlier broad speculative proposal. The earlier version attempted to combine emergent spacetime, de Sitter holography, Standard Model embedding, higher-spin fields, anomaly cancellation, and a single "entanglement field" explaining both dark matter and dark energy. That was too broad. The present version deliberately narrows the claim.

The central thesis is:

> *Einstein gravity is the first-order equation of local entanglement equilibrium. The dark sector may be the second-order effective response of local entanglement disequilibrium. In the minimal two-dimensional modular-response sector, the quantum geometric tensor induces a complex polar field. Its radial mode behaves as dark energy; its compact phase behaves as ultralight dark matter.*

The manuscript makes a conditional argument. It does not prove that the universe must contain this response sector. It proves that, given a set of physically motivated assumptions, the resulting dark-sector EFT has a constrained structure and non-trivial observational consequences.

1.1 What is imported, what is new, and what is conjectural

Component	Status	Role in this paper
Entanglement entropy and area	Established in black-hole thermodynamics and holography	Motivates generalized entropy and geometric entropy
Entanglement-equilibrium route to Einstein equations	Existing result, especially Jacobson-type causal-diamond arguments	Imported as first-order foundation
First law of entanglement for balls	Standard in CFT and holography	Connects modular energy to matter stress energy
Second-order relative entropy as positive Fisher information	Standard quantum information; holographically linked to canonical energy	Supplies the response metric
Modular Berry curvature	Existing but developing topic	Motivates complex structure in the light response sector
Two-dimensional light response sector	Conjectural model assumption	Gives the minimal polar field
Radial mode as dark energy	Derived within the EFT under slow roll	Observable through $w(z)$ and expansion history
Phase mode as ultralight dark matter	Derived within the EFT after coherent oscillation	Observable through structure formation and isocurvature
Fixed dark-sector coupling	Derived from polar field-space geometry	Main phenomenological novelty

The proposed novelty is not ultralight dark matter by itself, nor quintessence by itself, nor a complex scalar by itself. All of those exist in the literature. The novelty is the chain

relative entropy at second order → positive information metric
→ modular/Berry complex structure → polar entanglement-response field
→ coupled dark energy and phase dark matter.

2 First-order entanglement equilibrium and Einstein gravity

We set $\hbar = c = 1$ unless otherwise stated. Consider a small ball-shaped spatial region $B_R(p)$ of radius R centred at a spacetime point p, with the associated causal diamond. The generalized entropy is

$$S_{\text{gen}}(B_R) = \frac{A(\partial B_R)}{4G} + S_{\text{matter}}(B_R), \tag{1}$$

where $A(\partial B_R)$ is the area of the boundary of the ball and S_{matter} is the entanglement entropy of quantum fields restricted to the ball.

The entanglement-equilibrium hypothesis asserts stationarity of generalized entropy at fixed volume:

$$\delta S_{\text{gen}}\big|_V = 0. \tag{2}$$

Jacobson showed that, for first-order variations of small geodesic balls in a locally maximally symmetric vacuum state, this stationarity condition is equivalent to the semiclassical Einstein equation, with caveats for non-conformal matter sectors [5].

2.1 Local coefficient check in four dimensions

For a CFT vacuum reduced to a ball of radius R, the modular Hamiltonian is local:

$$K_B = 2\pi \int_{r<R} \mathrm{d}^3x\, \frac{R^2 - r^2}{2R} T_{00}(x). \tag{3}$$

If T_{00} is approximately constant over the ball,

$$\delta\langle K_B\rangle = 2\pi\delta T_{00}\, 4\pi \int_0^R r^2 \frac{R^2 - r^2}{2R} \mathrm{d}r = \frac{8\pi^2 R^4}{15}\delta T_{00}. \tag{4}$$

The entanglement first law gives

$$\delta S_{\text{matter}} = \delta\langle K_B\rangle. \tag{5}$$

At fixed volume, the leading geometric area variation of a small geodesic ball is

$$\delta A\big|_V = -\frac{4\pi R^4}{15}\delta G_{00}, \tag{6}$$

where G_{00} is the local Einstein tensor component in the freely falling frame. Substituting Eqs. (4) and (6) into Eq. (2) gives

$$0 = -\frac{\pi R^4}{15G}\delta G_{00} + \frac{8\pi^2 R^4}{15}\delta T_{00}, \tag{7}$$

and hence

$$\delta G_{00} = 8\pi G \delta T_{00}. \tag{8}$$

Demanding this relation for every local Lorentz frame yields

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{9}$$

with Λ appearing as the integration constant allowed by the Bianchi identity and stress-energy conservation.

This section is not claimed as a new derivation; it fixes notation and makes explicit the first-order foundation on which the second-order proposal rests.

3 Second-order relative entropy as entanglement disequilibrium

The quantum relative entropy of a state ρ with respect to a reference state ρ_0 is

$$S(\rho||\rho_0) = \mathrm{Tr}\rho\log\rho - \mathrm{Tr}\rho\log\rho_0 = \Delta\langle K\rangle - \Delta S, \tag{10}$$

where $K = -\log\rho_0$. It obeys

$$S(\rho||\rho_0) \geq 0. \tag{11}$$

For a one-parameter family of states $\rho(\lambda)$ around ρ_0, the first variation vanishes,

$$\delta S(\rho||\rho_0)\big|_{\lambda=0} = 0, \tag{12}$$

and the second variation defines a positive information metric,

$$S(\rho(\lambda)||\rho_0) = \frac{1}{2}\chi_{AB}\lambda^A\lambda^B + \mathcal{O}(\lambda^3), \qquad \chi_{AB} \geq 0. \tag{13}$$

In holographic CFTs, this second-order Fisher-information metric is dual to gravitational canonical energy for the corresponding wedge [7]. First-order relative entropy constraints are equivalent to linearized gravitational equations in the bulk [6]; second-order relative entropy therefore supplies a positive energetic measure of departure from an allowed equilibrium geometry.

Definition 1 (Local entanglement-disequilibrium residual)**.** *For a small causal diamond $D_R(x)$, define a generalized residual*

$$\mathcal{I}_R(x) = \Delta\langle K_R\rangle - \Delta S_{\mathrm{matter}}(D_R) - \Delta\left(\frac{A(\partial D_R)}{4G}\right), \tag{14}$$

where the first-order part vanishes on Einstein-equilibrium backgrounds:

$$\mathcal{I}_R^{(1)}(x) = 0. \tag{15}$$

The leading non-zero contribution is second order:

$$\mathcal{I}_R^{(2)}(x) = \frac{1}{2}\chi_{AB}^{(R)}(x)q^A(x)q^B(x) + \cdots, \qquad \chi_{AB}^{(R)} \geq 0. \tag{16}$$

Coarse-graining over a range of causal-diamond scales gives

$$\mathcal{F}_E = \int \mathrm{d}R\, W(R) \int \mathrm{d}^4x\sqrt{-g}\,\mathcal{I}_R^{(2)}(x). \tag{17}$$

At wavelengths much larger than the coarse-graining scale, locality, covariance, and positivity lead to an EFT of collective coordinates q^A:

$$\mathcal{F}_E = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}\mathcal{G}_{AB}(q)\nabla_\mu q^A\nabla^\mu q^B + U(q) + \cdots\right]. \tag{18}$$

The metric $\mathcal{G}_{AB}$ is interpreted as the coarse-grained information metric inherited from second-order relative entropy. This is the first key move: the dark sector is not introduced as extra matter by hand, but as the positive cost of local entanglement disequilibrium.

4 Quantum geometric tensor and the emergence of a complex response field

The real Fisher-information metric need not be the full geometric data of a family of quantum states or modular Hamiltonians. The quantum geometric tensor has the schematic form

$$\mathcal{Q}_{AB} = \chi_{AB} + i\omega_{AB}, \tag{19}$$

where χ_{AB} is real and symmetric, while ω_{AB} is antisymmetric and encodes Berry curvature or modular Berry curvature. Modular Berry connections have been shown to encode geometric information in holographic settings, including curve lengths and, in entanglement-wedge language, curvature and frame-sewing data [8, 9]. Recent operator-algebraic work further links modular Berry phases to emergent symplectic structure in large-N settings [10].

Conjecture 1 (Minimal modular-response sector)**.** *The lightest long-wavelength entanglement-disequilibrium sector is two-dimensional and carries a non-degenerate modular Berry curvature compatible with the positive Fisher metric.*

In a local orthonormal basis of the two response coordinates (q^1, q^2), write

$$\chi_{AB} = \delta_{AB}, \qquad \omega_{AB} = \kappa \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad \kappa \neq 0. \tag{20}$$

Then

$$J^A{}_B = \frac{1}{\kappa}\chi^{AC}\omega_{CB} \tag{21}$$

satisfies

$$J^2 = -\mathbf{1}. \tag{22}$$

Thus the light sector admits a natural complex structure. The corresponding complex coordinate is

$$\Phi_E = q^1 + iq^2 = \varrho e^{i\theta}. \tag{23}$$

The compact angular coordinate θ is the phase associated with this local complex structure; if the relevant modular/Berry holonomy closes, then

$$\theta \sim \theta + 2\pi. \tag{24}$$

This compactness is the origin of the axion-like behaviour in the phase sector.

Remark 1. *This is the most important strengthening relative to earlier versions of the proposal. The complex field is no longer chosen merely because a complex scalar is useful for cosmology. It arises as the natural coordinate on a two-dimensional second-order response space equipped with a Fisher metric and modular Berry curvature.*

5 Low-energy action and stability

The minimal covariant action for the response field is

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2}R - \frac{1}{2}(\nabla\varrho)^2 - \frac{1}{2}\varrho^2(\nabla\theta)^2 - U(\varrho) - \mu^4\left(1-\cos\theta\right)\right] + S_{\mathrm{vis}}[g_{\mu\nu}, \psi]. \tag{25}$$

Here ϱ has mass dimension one, θ is dimensionless, and μ is the small explicit-breaking scale for the compact phase. The visible sector is assumed to couple minimally to the metric at leading order. Direct non-gravitational portals are optional and constrained.

The field-space metric is

$$ds^2_{\text{field}} = d\varrho^2 + \varrho^2 d\theta^2. \tag{26}$$

This metric is positive for $\varrho > 0$, so the EFT has no ghost in the response sector. The phase potential is bounded below,

$$\mu^4(1 - \cos\theta) \geq 0. \tag{27}$$

The radial potential $U(\varrho)$ must be shallow enough for late-time slow roll and bounded from below in the EFT domain. A representative controlled choice is

$$U(\varrho) = U_0 + \Lambda_E^4 \left[1 - \exp\left(-\lambda \frac{\varrho - \varrho_0}{M_{\text{Pl}}}\right)\right]^2, \tag{28}$$

though the central coupling results below do not require this exact form.

The observed dark-energy scale fixes

$$\Lambda_E = \left(3M_{\text{Pl}}^2 H_0^2 \Omega_{\text{de}}\right)^{1/4} \simeq 2.2\,\text{meV}, \tag{29}$$

for Planck-normalised late-time cosmological parameters. In this framework, that meV scale is interpreted as the stiffness scale of long-wavelength entanglement disequilibrium, not as a bare sum over quantum-field vacuum energies.

5.1 Technical naturalness

The compact phase is protected by an approximate shift symmetry,

$$\theta \to \theta + \alpha, \tag{30}$$

softly broken by $\mu^4(1 - \cos\theta)$. This makes an ultralight phase technically natural in the usual axion-like sense: radiative corrections to m_θ are controlled by the symmetry-breaking spurion.

The radial mode is more delicate. A quintessence-like field with mass of order H_0 is radiatively unstable if it couples appreciably to visible matter. Therefore the minimal model assumes that ϱ is a dark entanglement-response coordinate with only gravitational-strength visible-sector coupling. This is not an optional aesthetic condition; it is required by fifth-force constraints and radiative stability.

6 Cosmological dynamics

Take a flat FRW metric with scale factor $a(t)$. The energy densities and pressures are

$$\rho_\varrho = \frac{1}{2}\dot{\varrho}^2 + U(\varrho), \qquad p_\varrho = \frac{1}{2}\dot{\varrho}^2 - U(\varrho), \tag{31}$$

$$\rho_\theta = \frac{1}{2}\varrho^2\dot{\theta}^2 + \mu^4(1 - \cos\theta), \qquad p_\theta = \frac{1}{2}\varrho^2\dot{\theta}^2 - \mu^4(1 - \cos\theta). \tag{32}$$

The Friedmann equation is

$$3M_{\text{Pl}}^2 H^2 = \rho_b + \rho_r + \rho_\varrho + \rho_\theta. \tag{33}$$

6.1 Radial mode as dynamical dark energy

The radial equation of state is

$$w_\varrho = \frac{p_\varrho}{\rho_\varrho} = \frac{\frac{1}{2}\dot{\varrho}^2 - U}{\frac{1}{2}\dot{\varrho}^2 + U}. \tag{34}$$

If

$$\dot{\varrho}^2 \ll U(\varrho), \tag{35}$$

then

$$w_\varrho \simeq -1. \tag{36}$$

Thus the radial entanglement-response mode behaves as dark energy, with small departures from -1 determined by its rolling.

6.2 Phase mode as ultralight dark matter

For small angular displacement,

$$\mu^4(1 - \cos\theta) \simeq \frac{1}{2}\mu^4\theta^2. \tag{37}$$

The local canonical phase field is

$$a_\theta = \varrho\theta, \tag{38}$$

so the phase mass is

$$m_\theta(\varrho) = \frac{\mu^2}{\varrho}. \tag{39}$$

When

$$m_\theta \gg H, \tag{40}$$

the phase oscillates coherently. In the harmonic regime, the oscillation-averaged pressure vanishes:

$$\langle p_\theta \rangle \simeq 0. \tag{41}$$

The phase therefore behaves as cold matter. This is the usual misalignment logic for ultralight scalar or axion-like dark matter, now interpreted as the compact phase of an entanglement-response field [13, 12].

7 Fixed dark-sector coupling from polar information geometry

The strongest phenomenological consequence comes from Eq. (39). For adiabatic phase oscillations, the comoving number density of phase quanta is conserved,

$$n_\theta \propto a^{-3}. \tag{42}$$

Since $\rho_\theta = m_\theta n_\theta$, Eq. (39) gives

$$\rho_\theta(a) = \rho_{\theta 0} a^{-3} \frac{m_\theta(a)}{m_{\theta 0}} = \rho_{\theta 0} a^{-3} \frac{\varrho_0}{\varrho(a)}. \tag{43}$$

Therefore

$$\dot{\rho}_\theta + 3H\rho_\theta = -\frac{\dot{\varrho}}{\varrho}\rho_\theta. \tag{44}$$

The radial equation can be written as

$$\ddot{\varrho} + 3H\dot{\varrho} + U_{,\varrho} = \frac{\rho_\theta}{\varrho}, \tag{45}$$

where the right-hand side is the averaged force from the angular motion. Multiplying Eq. (45) by $\dot{\varrho}$ gives

$$\dot{\rho}_\varrho + 3H(\rho_\varrho + p_\varrho) = +\frac{\dot{\varrho}}{\varrho}\rho_\theta. \tag{46}$$

Equations (44) and (45) conserve total dark-sector energy.

Proposition 1 (Geometric coupling)**.** *In the minimal polar entanglement-response model, the dark matter-dark energy interaction is fixed by the field-space metric and the phase mass drift. It is not a freely chosen phenomenological coupling.*

The resulting late-time consistency triangle is

$$\boxed{w_\varrho(a) \quad\Longleftrightarrow\quad \frac{\dot{\varrho}}{\varrho} \quad\Longleftrightarrow\quad \frac{\mathrm{d}\ln m_\theta}{\mathrm{d}\ln a} \quad\Longleftrightarrow\quad \dot{\rho}_\theta + 3H\rho_\theta.} \tag{47}$$

In particular,

$$\frac{\mathrm{d}\ln m_\theta}{\mathrm{d}\ln a} = -\frac{\mathrm{d}\ln \varrho}{\mathrm{d}\ln a}. \tag{48}$$

A measurement of late-time dark-energy evolution therefore fixes a correlated deviation in the dark-matter dilution law, once the radial trajectory is specified.

8 Apparent phantom reconstruction without a ghost

Cosmological analyses often infer dark energy assuming that dark matter dilutes exactly as a^{-3}. But in this model the actual phase density follows Eq. (43). If an observer assumes standard CDM dilution, the inferred dark-energy density is

$$\rho_{\mathrm{DE}}^{\mathrm{inf}}(a) = \rho_\varrho(a) + \rho_{\theta 0} a^{-3}\left[\frac{\varrho_0}{\varrho(a)} - 1\right]. \tag{49}$$

The inferred equation of state is

$$w_{\mathrm{inf}}(a) = -1 - \frac{1}{3}\frac{\mathrm{d}\ln \rho_{\mathrm{DE}}^{\mathrm{inf}}}{\mathrm{d}\ln a}. \tag{50}$$

If $\rho_{\mathrm{DE}}^{\mathrm{inf}}$ increases with scale factor, then $w_{\mathrm{inf}} < -1$ even when the true radial field is canonical and non-phantom,

$$w_\varrho \geq -1. \tag{51}$$

The model can therefore generate apparent phantom-like reconstruction without ghost instability. This is not a guarantee that current data require such behaviour, but it is a useful way to interpret phenomenological $w_0 w_a$ fits that drift into phantom-like or crossing-like regions.

9 Inflationary consistency and isocurvature

If the compact phase exists during inflation and later constitutes a fraction F_θ of total dark matter, quantum fluctuations of the phase create cold-dark-matter isocurvature perturbations. For a homogeneous initial displacement $a_i = f_a \theta_i$,

$$\delta a = \frac{H_I}{2\pi}, \tag{52}$$

and for quadratic oscillations

$$S_\theta \simeq 2\frac{\delta a}{a_i} = \frac{H_I}{\pi f_a \theta_i}. \tag{53}$$

The total CDM isocurvature contribution is $F_\theta S_\theta$, so

$$\mathcal{P}_S \simeq F_\theta^2 \left(\frac{H_I}{\pi f_a \theta_i}\right)^2. \tag{54}$$

Using

$$H_I = \pi M_{\rm Pl}\sqrt{\frac{A_s r}{2}}, \tag{55}$$

and imposing $\mathcal{P}_S < \beta_{\rm iso} A_s$, one finds

$$r < 2\beta_{\rm iso}\left(\frac{f_a\theta_i}{F_\theta M_{\rm Pl}}\right)^2. \tag{56}$$

For the full-DM branch $F_\theta = 1$, the misalignment abundance gives, for oscillation defined by $3H(a_{\rm osc}) = m_\theta$ during radiation domination,

$$f_a|\theta_i| \simeq 2.7 \times 10^{16}\,\mathrm{GeV}\left(\frac{m_\theta}{10^{-20}\,\mathrm{eV}}\right)^{-1/4}. \tag{57}$$

Substituting into Eq. (56) gives

$$r_{\rm max} \simeq 9.4 \times 10^{-6}\left(\frac{\beta_{\rm iso}}{0.038}\right)\left(\frac{m_\theta}{10^{-20}\,\mathrm{eV}}\right)^{-1/2}\frac{1}{F_\theta}. \tag{58}$$

Thus the pre-inflationary full-phase-dark-matter branch predicts very small primordial tensors. Current tensor bounds, such as $r_{0.05} < 0.036$ from BICEP/Keck 2018, do not reach this level [17]. A future robust detection around $r \sim 10^{-3}$ would severely challenge or rule out this branch unless the phase is generated after inflation, contributes only a subdominant fraction, or has modified initial conditions.

m_θ [eV]	$z_{\rm osc}$	$f_a\|\theta_i\|$ [GeV]	$r_{\rm max}$ for $F_\theta = 1$
1×10^{-20}	1.6×10^7	2.7×10^{16}	9.4×10^{-6}
3×10^{-20}	2.7×10^7	2.1×10^{16}	5.4×10^{-6}
1×10^{-19}	4.9×10^7	1.5×10^{16}	3.0×10^{-6}

The numbers in the table use $H_0 = 67.4\,\mathrm{km\,s^{-1}Mpc^{-1}}$, $\Omega_r = 9.2 \times 10^{-5}$, and $\Omega_{\rm dm} = 0.265$. They are benchmarks, not fits.

10 Observational implications

10.1 Dark energy

The radial mode predicts either $w_\varrho \simeq -1$ or a small time-dependent departure. DESI DR2 reports that BAO plus CMB data prefer a w_0w_a dynamical dark-energy model over ΛCDM at 3.1σ, with the significance depending on the supernova sample when supernovae are included [18]. This is not evidence for the present model specifically. It is compatible pressure in the right observational direction: late-time dark energy may not be exactly a constant.

The model should be confronted with DESI, CMB, supernovae, weak lensing, and redshift-space distortion data through a Boltzmann-code implementation. Until then, DESI-like hints should be described as suggestive compatibility, not confirmation.

10.2 Ultralight phase dark matter

The phase mode belongs to the ultralight scalar/axion-like dark-matter class, not to the WIMP class. It should be tested through structure formation, lensing, CMB isocurvature, and specialised precision-field searches rather than ordinary nuclear-recoil experiments. Fuzzy dark matter was classically discussed around $m \sim 10^{-22}$ eV [11, 12], but canonical all-dark-matter models at such low masses are under pressure from Lyman-alpha forest data. Rogers and Peiris report a strong lower bound $m_a > 2 \times 10^{-20}$ eV for canonical ultra-light axion dark matter [14]. Therefore the safer benchmark region for the full-phase-DM branch is closer to

$$m_\theta \gtrsim \text{few} \times 10^{-20}\,\text{eV}, \tag{59}$$

with lower masses treated as stressed or subdominant branches.

10.3 Cosmic birefringence and optional portals

If the compact phase couples to electromagnetism,

$$\mathcal{L}_{\theta\gamma} = -\frac{g_{\theta\gamma}}{4} a_\theta F_{\mu\nu} \tilde{F}^{\mu\nu}, \tag{60}$$

then a changing phase rotates photon polarisation by

$$\Delta\beta = \frac{g_{\theta\gamma}}{2} \Delta a_\theta. \tag{61}$$

This offers a possible parity portal to cosmic birefringence. However, for masses $m_\theta \gtrsim 10^{-20}$ eV, the field oscillates rapidly on cosmological propagation times, and static birefringence explanations are not generic. Recent analyses of ultralight ALP dark matter in light of cosmic birefringence find strong washout and mass-window constraints [19]. Therefore birefringence is optional phenomenology, not a core pillar of the model.

10.4 What would count as supportive evidence

No single observation would uniquely establish the theory. The evidence would have to be a correlated pattern:

1. dark energy evolves with $w(z)$ close to, but distinguishable from, -1;
2. dark matter behaves as a real collisionless component but with ultralight/wave-like structure signatures;
3. the pre-inflationary branch has no detectable primordial tensors above the 10^{-5} level;
4. isocurvature constraints are satisfied in the same parameter region that fits the abundance;
5. the dark-matter dilution law and dark-energy rolling are consistent with Eq. (44);
6. any non-gravitational portal, such as birefringence, matches the same phase field rather than an unrelated axion-like particle.

11 Novelty relative to neighbouring models

Model class	Existing idea	Difference here
ΛCDM	Constant dark energy and collisionless CDM	No entanglement-response origin; no fixed radial/phase coupling
Quintessence	Slow-rolling scalar dark energy	Radial field is not independent; coupled to phase DM by polar geometry
Axion or fuzzy DM	Coherent ultralight scalar oscillations act as matter	Phase field is the compact angular coordinate of an entanglement-response order parameter
Spintessence / complex scalar cosmology	Complex scalar can have radial and angular cosmological dynamics	Here the complex structure is motivated by the second-order quantum geometric tensor
Interacting dark energy	Couplings Q are often phenomenological	Coupling follows from $m_\theta \propto \varrho^{-1}$ and the polar metric
Emergent gravity without real DM	Apparent dark matter arises from modified gravity	This model contains a real collisionless phase energy density, helping with lensing/cluster phenomenology

The model should not claim to be the first complex scalar dark sector. It should claim a specific information-geometric origin and a fixed late-time coupling relation.

12 How the model fails

A serious speculative theory must expose its failure modes. This model is weakened or falsified under the following conditions:

1. **Tensor detection.** A robust detection of primordial tensors at $r \sim 10^{-3}$ would rule out the pre-inflationary full-phase-DM branch unless the phase is generated after inflation or is subdominant.

2. **Constant dark energy.** If future DESI/Euclid/Roman/Rubin combinations pin $w(z) = -1$ with no allowed evolution, the radial-response interpretation loses motivation, though it may survive in an effectively frozen limit.

3. **Structure exclusion.** If Lyman-alpha, lensing, dwarf-galaxy, and high-redshift structure data exclude the required ultralight mass/fraction window, the phase-DM branch fails.

4. **WIMP discovery.** A confirmed conventional WIMP or other particle that accounts for all dark matter would strongly disfavor the phase-DM interpretation.

5. **No complex modular sector.** If no defensible modular/Berry or quantum-geometric derivation of a two-dimensional response sector can be made, the entanglement origin collapses into interpretation rather than mechanism.

6. **Radiative instability.** If the radial mode cannot be protected from visible-sector loops while producing observable dynamics, the model is technically unnatural.

13 Conditional theorem

Theorem 1 (Entanglement-disequilibrium dark-sector theorem)**.** *Assume:*

(A1) Local causal diamonds satisfy generalized entanglement equilibrium at fixed volume,

$$\delta S_{\text{gen}}|_V = 0.$$

(A2) The matter sector obeys the entanglement first law for small ball-shaped regions,

$$\delta S_{\text{matter}} = \delta\langle K\rangle.$$

(A3) The fixed-volume area variation of small causal diamonds is governed by local curvature as in Eq. (6).

(A4) The first non-vanishing departure from equilibrium is the positive second-order relative entropy, Eq. (13).

(A5) The lightest long-wavelength response sector is two-dimensional and carries non-degenerate modular Berry curvature compatible with the Fisher metric.

(A6) The compact angular coordinate is softly broken by a periodic potential and the radial mode is sufficiently sequestered from visible-sector radiative corrections.

Then:

(C1) First-order stationarity yields the semiclassical Einstein equation.

(C2) Second-order disequilibrium defines a positive low-energy response functional.

(C3) The two-dimensional response sector admits a natural complex coordinate $\Phi_E = \varrho e^{i\theta}$.

(C4) Under slow roll, the radial mode behaves as dynamical dark energy.

(C5) After coherent oscillation begins, the compact phase behaves as ultralight dark matter.

(C6) The dark matter-dark energy interaction is fixed by the polar information metric:

$$\dot{\rho}_\theta + 3H\rho_\theta = -\frac{\dot{\varrho}}{\varrho}\rho_\theta. \tag{62}$$

Proof. Assumptions A1–A3 reproduce the Jacobson-type causal-diamond derivation of the semiclassical Einstein equation, as shown explicitly in Section 2. A4 gives a positive quadratic functional for departures from equilibrium, which becomes a covariant long-wavelength response EFT after coarse-graining. A5 supplies the quantum-geometric data needed to define an almost complex structure on the light two-dimensional response sector; choosing polar coordinates gives $\Phi_E = \varrho e^{i\theta}$. A6 gives a stable compact phase potential and a technically controlled radial sector. The equations of motion from Eq. (25) then imply slow-roll dark-energy behaviour for ϱ, coherent matter-like oscillations for θ, and the adiabatic phase-density scaling Eq. (43). Differentiating that scaling gives the fixed coupling. □

14 Conclusion

The most robust version of the proposal is not a theory of everything. It is a conditional low-energy statement about what may follow after taking entanglement equilibrium seriously beyond first order.

The first-order statement is known: local entanglement equilibrium can reproduce Einstein gravity. The new proposal is that the second-order relative-entropy residual should not be discarded. It defines a positive response metric. If the lightest response sector has a modular/Berry complex structure, the natural field coordinate is polar. In that coordinate system, the radial mode is dark-energy-like, the compact phase is ultralight-dark-matter-like, and their coupling is fixed by information geometry.

The concise form is

$$\boxed{\text{Einstein gravity} = \text{first-order entanglement equilibrium}}$$

$$\boxed{\text{dark sector} = \text{second-order entanglement disequilibrium response}}$$

$$\boxed{\Phi_E = \varrho e^{i\theta}: \quad \varrho \to \text{dark energy}, \quad \theta \to \text{ultralight dark matter.}}$$

This framework earns its keep only if it produces correlated observational consequences. The most important are: a fixed coupling between radial rolling and phase-dark-matter dilution, very small primordial tensors in the pre-inflationary full-DM branch, viable ultralight structure signatures above current Lyman-alpha pressure, and possible but optional parity-violating portals. Future data can therefore strengthen, restrict, or kill the model.

A Dimensional checks

The field dimensions in Eq. (25) are

$$[\varrho] = 1, \qquad [\theta] = 0, \qquad [\mu] = 1. \tag{63}$$

The phase mass in the harmonic limit is

$$[m_\theta^2] = \left[\frac{\mu^4}{\varrho^2}\right] = 2, \tag{64}$$

as required. The coupling equation

$$\dot{\rho}_\theta + 3H\rho_\theta = -\frac{\dot{\varrho}}{\varrho}\rho_\theta \tag{65}$$

has dimension 5 on both sides in natural units: $[\dot{\rho}] = 5$, $[H\rho] = 5$, and $[\dot{\varrho}/\varrho] = 1$.

B Benchmark derivation for the misalignment amplitude

For constant phase mass during radiation domination, oscillation begins when

$$3H(a_{\rm osc}) = m_\theta, \qquad H(a) = H_0\sqrt{\Omega_r}\, a^{-2}. \tag{66}$$

Hence

$$a_{\rm osc} = \left(\frac{3H_0\sqrt{\Omega_r}}{m_\theta}\right)^{1/2}. \tag{67}$$

The present phase density is approximately

$$\rho_{\theta 0} = \frac{1}{2} m_\theta^2 (f_a \theta_i)^2 a_{\rm osc}^3. \tag{68}$$

Setting $\rho_{\theta 0} = 3M_{\rm Pl}^2 H_0^2 \Omega_{\rm dm}$ gives Eq. (57).

C Suggested numerical programme

A full test requires implementation in CLASS or CAMB with a Monte Carlo pipeline such as MontePython or Cobaya. The minimal parameter set is

$$\left\{\Omega_b h^2, \Omega_c h^2, H_0, A_s, n_s, \tau, m_\theta, \lambda, \varrho_0, U_0, F_\theta\right\}, \tag{69}$$

with priors chosen so that $\varrho > 0$, the radial sector is ghost-free, and the phase begins oscillating before matter-radiation equality for the full-DM branch. Observables include $H(z)$, $f\sigma_8(z)$, CMB TT/TE/EE spectra, lensing, BAO distances, the matter power spectrum, isocurvature, and any parity-violating photon portal.

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